Solutions to JEE Advanced Booster Test – 2 | 2024 | Code A

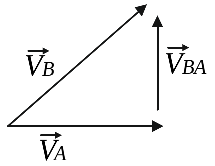
[PHYSICS]

1.(A) $\vec{V}_A = 15\ m/s(\hat{i})$

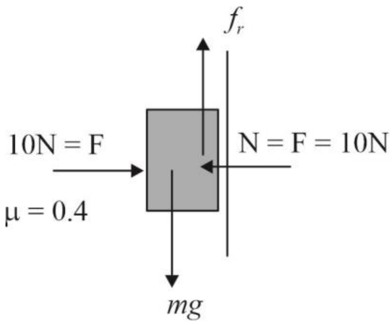
$\vec{V}_{BA} = 5\hat{j}\ m/s$

$\vec{V}_B = \vec{V}_{BA} + \vec{V}_A = 5\hat{j} + 15\hat{i}$

$|\vec{V}_B| = \sqrt{25+225} = 5\sqrt{10}\ m/s$



2.(A)



$f_r = \mu N = (0.4)(10)$ Newton

$f_r = 4$ newton

weight $(mg) = f_r = 4N$

3.(A)

4.(B) At $t = 0$ sec

Velocity of ant wrt. Stick $(u_{AS}) = u\,(\uparrow)$

Velocity of stick wrt ground $(u_{SG}) = \frac{u}{2}(\downarrow)$

Acceleration of stick wrt ground $(a_{SG}) = g\,(\downarrow)$

Acceleration of ant wrt stick $(a_{AS}) = 0$

In reference frame of ground,

$u_{AG} = u_{AS} + u_{SG} = u - \frac{u}{2} = \frac{u}{2}$

$a_{AG} = a_{AS} + a_{SG} = 0 + (-g) = -g$

For attaining max height $V_{AG} = 0$

$V^2{}_{AG} = u^2{}_{AG} + 2(a_{AG})(H)$

$\Rightarrow \quad H = \frac{u^2}{8g}$

For returning to initial point displacement of ant wrt groud $x_{AG} = 0$

$x_{AG} = u_{AG}T + \frac{1}{2}(a_{AG})T^2 \qquad \Rightarrow T = \frac{u}{g}$

5.(A) $x = 3t^3 - 18t^2 + 36t$

$V = 9t^2 - 36t + 36$

$V = 9(t^2 - 4t + 4)$

$V = 9(t-2)^2$ But its sign does not change

$V = 0$ at $t = 2$

So distance = |displacement|

$a = \dfrac{dv}{dt} = 18(t-2)$

At $t = 0,\ a = 0$

6.(ABD) $v_1 = \dfrac{s_1}{t_2 - t_1} = 25\text{m/s}$

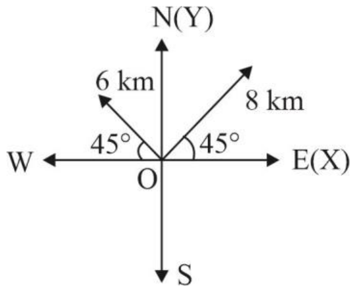
$v_2 = \dfrac{s_2}{t_2 - t_1} = 10\,\text{m/s}$

$l = \dfrac{s_2(t_3 - t_1)}{(t_2 - t_1)} = 700\text{ m}$

The front and the rear cars travel the distances s_2 and s_1 respectively in the interval $[t_1, t_2]$.

The front and the rear cars spend the intervals $[t_1, t_3]$ and $[t_2, t_4]$ respectively on the bridge.

7.(AD)



(A) Net movement along x-direction,

$$S_x = (8-6)\cos 45° = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}\text{ km}$$

Net movement along y-direction,

$$S_y = (8+6)\sin 45° = 14 \times \frac{1}{\sqrt{2}} = 7\sqrt{2}\text{ km}$$

Net movement from starting point,

$$|\vec{S}| = \sqrt{S_x^2 + S_y^2} = \sqrt{(\sqrt{2})^2 + \left(7\sqrt{2}\right)^2} = 10\text{ km}$$

(D) Angle which makes with the east direction,

$$\tan\theta = \frac{Y - \text{component}}{X - \text{component}} = \frac{7\sqrt{2}}{\sqrt{2}} \therefore \theta = \tan^{-1}(7)$$

8.(ACD)

$t_{\min} = \dfrac{b}{V_{mr}} = 60$ ...(1)

For shortest path, $t = \dfrac{b}{\sqrt{V_{mr}^2 - V_r^2}} = 180$ ...(2)

(A) $V_{mr} > V_r$

(C) $V_{mr} = \dfrac{b}{60\,\text{min}} = \dfrac{3\sqrt{2}km}{1\text{ hr}} = 3\sqrt{2}\text{ km/h}$

By (2), $\sqrt{V_{mr}^2 - V_r^2} = \sqrt{2}$ km/h

$\Rightarrow V_r = \sqrt{V_{mr}^2 - 2} = \sqrt{(3\sqrt{2})^2 - 2} = 4$ km/h

(D) $V_{mr} = 3\sqrt{2}km/h, V_r = 4km/h$

$$t = \frac{b}{V_{mr}\sin\theta} \Rightarrow \sin\theta = \frac{3\sqrt{2}km}{3\sqrt{2}\text{km}\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°$$

Drift $= x = (V_r - V_{mr}\cos 45°)t = \sqrt{2}$ km

9.(BD)

(B) $S = ut + \frac{1}{2}at^2$

$$S_1 = t + \frac{t^2}{2}$$

$$S_2 = 2t + t^2$$

$$S_3 = 3t + \frac{3}{2}t^2$$

$$S_4 = 4t + \frac{4}{2}t^2$$

$\vdots$

$$\left.\begin{aligned} S_2 - S_1 &= t + \frac{t^2}{2} \\ S_3 - S_2 &= t + \frac{t^2}{2} \\ S_4 - S_3 &= t + \frac{t^2}{2} \end{aligned}\right\} \text{ same for two consecutive particles}$$

(D) Let $S = t + \frac{t^2}{2}$

$\frac{dS}{dt} = 1 + t,\ \frac{d^2S}{dt^2} = 1 =$ constant

$\therefore$ S increases periodically with time.

10.(ABC)

(A) $v = \frac{dx}{dt} = ax + b$

$$\Rightarrow \int_0^x \frac{dx}{ax+b} = \int_0^t dt$$

$$\Rightarrow \log_e|ax+b| - \log|b| = at$$

$$\Rightarrow \log_e\left|\frac{ax+b}{b}\right| = at$$

$$\Rightarrow ax + b = be^{at}$$

$$\therefore x = \frac{b}{a}\left(e^{at} - 1\right)$$

(B) $v = ax + b = be^{at}$

$$A = \frac{dv}{dt} = abe^{at}$$

(C) $v = be^0 = b$ at $t = 0$

11.(C) Applying $S = ut + \frac{1}{2}at^2$ for the body

$$h - 120.5 = 4 \times 5 + \frac{1}{2} \times (-9.8)25$$

$$h = 120.5 + 20 - 4.9 \times 25 \;;\; h = 18\ m$$

12.(A) For balloon $h - 120.5 = 4 \times 5$ $(s = ut)$

$$h = 140.5$$

Separation $= (140.5 - 18)\ m = 122.5\ m$

SECTION 2

1.(0.35) $V_{br} = 4V_r$ time taken $= t = \dfrac{d}{\sqrt{V_{br}^2 - V_r^2}}$

$$V_{br}' = \beta V_{br}$$

$$\therefore 1 = \frac{d}{\sqrt{V_{br}^2 - V_r^2}} = \frac{d}{\sqrt{16V_r^2 - V_r^2}} = \frac{d}{\sqrt{15}V_r} \quad ...(1)$$

$$\therefore 4 = \frac{d}{\sqrt{\beta^2 V_{br}^2 - V_r^2}} = \frac{d}{\left(\sqrt{16\beta^2 - 1}\right)V_r} \quad ...(2)$$

Equate, (1) and (2) $\sqrt{15}V_r = 4\sqrt{16\beta^2 - 1}V_r$

$$\Rightarrow 15 = 16(16\beta^2 - 1) \qquad \Rightarrow 16\beta^2 = \frac{31}{16}$$

$$\therefore \beta = \frac{\sqrt{31}}{16} = 0.35$$

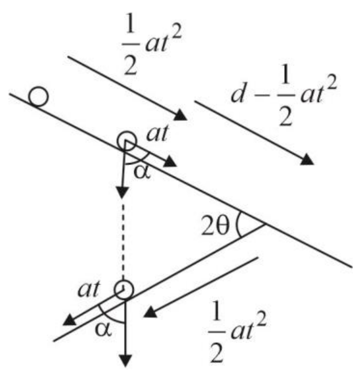
2.(8) $\frac{1}{2} g \sin\theta \times (12)^2 = \frac{1}{2} g \sin\theta (4)^2 + d$ (Here d is the initial separation between A and B)

$[\because a = g\sin\theta]$

$$\Rightarrow 72a = 8a + d$$

$$\Rightarrow 64a = d \quad ...(1)$$

They will be closest to each other, if both the balls are in same vertical line

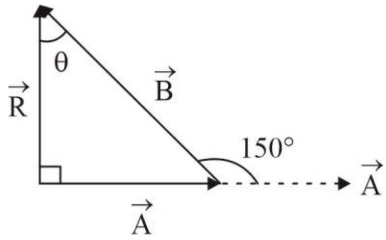


$\therefore\ d-\frac{1}{2}at^2=\frac{1}{2}at^2 \quad \Rightarrow \quad at^2=d \quad \therefore \quad t=\sqrt{\frac{d}{a}}=\sqrt{\frac{64a}{a}}=8$ sec

3.(150) $R=B\cos\theta.$ As per question,

$R=B/2=B\cos\theta$

or $\cos\theta=1/2$ or $\theta=60°$



Hence, angle between $\vec{A}$ and $\vec{B}=90°+60°=150°$

4.(5.06) Let ball reach the ground for the first time with speed $v=\sqrt{2gh}=\sqrt{2\times10\times20}=20\,m/s$

Total time $=\frac{V}{g}+\left[\frac{2V(0.8)}{g}+\frac{2V(0.8)^2}{g}+---\right]=\frac{V}{g}\left[1+2(0.8+(0.8)^2+----)\right]=2\left[1+2\left(\frac{0.8}{1-0.8}\right)\right]$

$=2[1+8]=18$sec.

Total distance $=h+2\left[\frac{v^2(0.8)^2}{2g}+\frac{v^2(0.8)^4}{2g}+......\right]=h+\frac{v^2}{g}[0.64+(0.64)^2+...]$

$=20+40\left[\frac{0.64}{1-0.64}\right]=20+40\left(\frac{16}{9}\right)=\frac{820}{9}m \quad \Rightarrow \quad V_{av}=\frac{820}{9(18)}=5.06$

5.(10) After $t=4s$, velocity of B becomes lesser than velocity of A and distance between the particles start decreasing. Separation is maximum at $t=4s$

$d_{\max}=(X_A \text{ at } t=4)-(X_B \text{ at } t=4)$

$=\frac{1}{2}\times4\times10-\frac{1}{2}\times2\times10=10m$

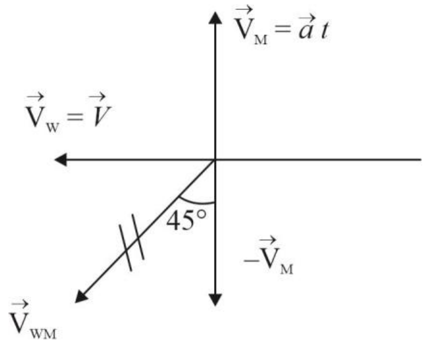
6.(1)

$\tan 45°=\frac{V_W}{V_M}$

$\Rightarrow V_M=V_W$

$\Rightarrow at=v$

$t=\frac{v}{a}\therefore\ n=1$



[CHEMISTRY]

1.(A) 6 mole HCl $\equiv$ 3 mole $H_2 = 3 \times 22.4$ litre H_2.

2.(D) Bohr's model was successfully applied to single electron system, e.g., H, He^+, Li^{2+}, etc.

3.(D) $4 \times m_p = m_\alpha$

$\therefore \quad \frac{m_p}{m_\alpha} = \frac{1}{4}$

Also, $\frac{\lambda_p}{\lambda_\alpha} = \frac{h}{m_p.v_p} \times \frac{m_\alpha.v_\alpha}{h} = \frac{4}{1} \times \frac{v_\alpha}{v_p} = \frac{1}{2}$

$\therefore \quad \frac{v_p}{v_\alpha} = 8:1 \quad \Rightarrow \quad \frac{KE_p}{KE_\alpha} = \frac{\frac{1}{2} \times m_p \times v_p^2}{\frac{1}{2} \times m_\alpha.v_\alpha^2} = \frac{1}{4} \times 8^2 = 16:1$

4.(B) meq. of CO_2 = meq. of NaOH

$= \frac{20}{40} \times 1000 = 500$ (valence factor of $CO_2 = 2$)

$\therefore$ mole of $CO_2 = \frac{500}{2} \times \frac{1}{1000} = 0.25$

$\therefore$ mole of CO = 0.75

On complete oxidation of CO to CO_2, 0.75 mole of CO_2 are formed

$\therefore$ mole of CO_2 formed as a result of oxidation of CO = 0.75

$\therefore$ eq. of $CO_2 = 0.75 \times 2 = 1.50$

$\therefore$ eq. of NaOH required for this $CO_2 = 1.50$

$\frac{w}{40} = 1.50 \quad \Rightarrow \quad w = 60g$

5.(C) Number of nodal planes = 2 (excluding one of origin)

$\therefore \quad n - l - 1 = 5 - l - 1 = 2$

$\therefore \quad l = 2$ i.e., d-orbital (observation ii)

$\therefore \quad l = 2$

Observation (i) leads for 3d, 4d or 5d however 5d has two nodal plane, 4d has one and 3d has none. Now xy is nodal plane, thus orbital is yz (observation iii)

6.(ABCD) $55.2 = \frac{46x + (1-x)92}{1}$

$x = 0.8$ mole

% mole = $\frac{0.8}{1} \times 100 = 80\%$

$X_{N_2O_4} = 0.2$

Mole of N = 0.8 + 0.4 = 1.2

Mole % of N = $\frac{1.2}{1} \times 100 = 120$

Wt % $N_2O_4 = \frac{20 \times 92}{55.2} = 33.3\%$

7.(ACD)

(A) $2NaHCO_{3(s)} \longrightarrow Na_2CO_{3_{(s)}} + H_2O_{(g)} + CO_{2_{(g)}}$

$$n \qquad \frac{n}{2} \qquad \frac{n}{2} \qquad \frac{n}{2}$$

$MgCO_{3_{(s)}} \longrightarrow MgO_{(s)} + CO_{2_{(g)}}$

$$n \qquad n \qquad n$$

$n_{gases} = \frac{n}{2} + \frac{n}{2} + n = \frac{1 \times 896}{0.0821 \times 546} \Rightarrow 20$

$\therefore \quad n = 10$

Mass of $MgCO_3 = 840gm$

Mass of sample $\times \frac{84}{100} = 1680g$

$M_{sample} = 2000gm.$

(B) Mass of impurity = $2000 \times \frac{16}{100} = 320g$

(C) $M_{avg} = \frac{5 \times 18 + 15 \times 44}{20} = 37.5$

(D) Mass of residue = $2000 - 750 = 1250g$

8.(ABC)

$4.25 = W_A + KE_A \qquad \ldots.(1)$

$4.2 = W_B + KE_B \qquad \ldots.(2)$

Given, $KE_B = KE_A - 1.5$

$\lambda_B = 2\lambda_A \quad \Rightarrow \quad \frac{h}{\sqrt{2mKE_B}} = \frac{2h}{\sqrt{2mKE_A}}$

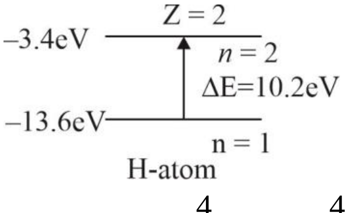
$KE_B = \frac{KE_A}{4} = KE_A = 4KE_B$

$KE_B = 0.5eV$

$KE_A = 2eV$ and $W_A = 2.25eV$

$W_B = 3.7$ eV

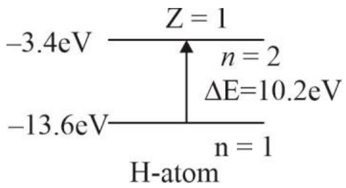
9.(ABD)



$\Delta E = -13.6 \times \frac{4}{16} + 13.6 \times \frac{4}{4} = 13.6 - 3.4$

$\Delta E = 10.2eV$

(A)



$$2\pi r = n\lambda \text{ for } n = 2$$

$$2\pi\left(0.529\times\frac{4}{1}\right)\text{Å} = 2\times\lambda$$

(B) Ionization energy = + 3.4 eV

(C) $\lambda = \pi\times 0.529\times 4$

(D) $\frac{\lambda}{\pi a_0} = \frac{\pi\times 0.529\times 4}{\pi\times 0.529} = 4$

10.(BC)

(A) $6-1=5$

(B) $6-2=4$

(C) $(6-3)+(6-4)+(6-5) = 3+2+1=6$

(D) Max. number of lines $= \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = 15$

11.(C) Separation energy $= E_\infty - E_n$

$$13.6 = 0+13.6\times\frac{z^2}{(3)^2} \quad \therefore \quad z = 3$$

$$IE = 13.6\,z^2$$

$$IE = 13.6\times(3)^2 = 122.4\text{eV}$$

12.(B) $\frac{1}{\lambda} = R\times 3^2\left[\frac{1}{2^2}-\frac{1}{\infty^2}\right] = \frac{9R}{4}$

$$\lambda = \frac{4}{9R}$$

SECTION 2

1.(4) Statements 1, 2, 4 and 5 are correct and 3 and 6 are incorrect.

2.(7) $n\times l$ = zero, if $l = 0\,(s)$

So total number of electrons in Cr = 7

3.(5) Sixth excited state of He is 3d and its degeneracy is 5.

4.(5)

$$3Hg + 2Cl_2 \longrightarrow Hg_2Cl_2 + HgCl_2$$

	3Hg	2Cl₂	Hg₂Cl₂	HgCl₂
	3 moles	2 moles	1 mole	1 mole
Given number of moles =	0.75	0.5		

moles of product formed = $\frac{0.75}{3} = 0.25$

So total moles of product formed = 0.25 + 0.25 = 0.50

So x = 5

5.(4) Speed $= \dfrac{\text{Distance}}{\text{time}} \quad \Rightarrow \quad \dfrac{1}{t} = \dfrac{V_n}{2\pi r_n}$

Number of revolution per second $= \dfrac{V_n}{2\pi r_n}$

$\because$ Number of revolution $\propto \dfrac{z^2}{n^3}$

6.(238) $\dfrac{19.7}{197} \times 119 = 11.9$

$11.9 \times \dfrac{1}{0.4} \times \dfrac{1}{0.5} \times \dfrac{1}{0.25} = 238$

[MATHEMATICS]

SECTION 1

1.(D) $x^{\ln x^2} = e^{1018}$

$\Rightarrow \ln x^{\ln x^2} = \ln e^{2018}$

$\Rightarrow 2(\ln x)^2 = 2018 \Rightarrow x_1 = e^{\sqrt{1009}}, x_2 = e^{-\sqrt{1009}}$

(A) $\dfrac{\cot^2 5°}{\operatorname{cosec}^2 5° - 1} = 1$

(B) $\tan\theta.\tan(60° - \theta).\tan(60° - \theta) = \tan 3\theta$

(C) $\cos 27° = 4\cos^3 9° - 3\cos 9°$

$\dfrac{4\cos^3 9° - \cos 27°}{3\cos 9°} = 1$

and $\sin 27° = 3\sin 9° - 4\sin^3 9°$

$\dfrac{4\sin^3 9° + \sin 27°}{3\sin 9°} = 1$

(D) $\dfrac{4\sin 40° \sin 50°}{\cos 80°} \dfrac{\sin 10°}{\cos 10°} = 2$

2.(C) $A_k = \dfrac{K(K-1)}{2}\cos\dfrac{K(K-1)\pi}{2}$

$A_{19} = \dfrac{19\times 18}{2}\cos\dfrac{19\times 18\pi}{2} = \dfrac{19\times 18}{2}\cos 19\times 9\pi = -\dfrac{19\times 18}{2}$

$A_{20} = \dfrac{20\times 19}{2}\cos\dfrac{20\times 19\pi}{2} = \dfrac{20\times 19}{2}\cos 10\times 19\pi = \dfrac{20\times 19}{2}$

$A_{21} = \dfrac{21\times 20}{2}\cos\dfrac{20\times 21\pi}{2} = \dfrac{21\times 20}{2}$

$A_{22} = \dfrac{22\times 21}{2}\cos\dfrac{22\times 21\pi}{2} = -\dfrac{22\times 21}{2}$

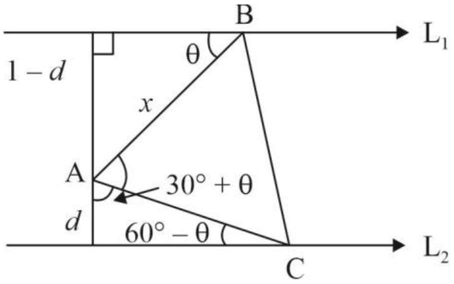
$$\left.\begin{aligned} A_{23} &= \frac{23\times 22}{2}(-1) \\ A_{24} &= \frac{24\times 23}{2}(1) \\ A_{25} &= \frac{25\times 24}{2}(1) \\ A_{26} &= \frac{26\times 25}{2}(-1) \end{aligned}\right\} \begin{aligned} & A_{19}+A_{20}+A_{21}+A_{22} = -2 \\ & \text{value of } \cos\frac{K(K-1)\pi}{2} \\ & \text{will repeat after four terms} \end{aligned}$$

$$A_{23}+A_{24}+A_{25}+A_{26} = -2$$

Similarly Group of 4 terms will give answer – 2

so $\left|A_{19}+A_{20}+....A_{98}\right| = |-40| = 40$

3.(B)



$x\cos(\theta+30°) = d$ and $x\cos\left(90°-\theta\right) = 1-d$

$x\sin\theta = 1-d$

Dividing $\sqrt{3}\cot\theta = \dfrac{1+d}{1-d}$, squaring eq. (ii) and putting the value of $\cot\theta$, we get

$$x^2 = \frac{1}{3}(4d^2-4d+4)$$

4.(B)

$$x = \frac{\sum_{n=1}^{44}\cos n°}{\sum_{n=1}^{44}\sin n°}$$

$$= \frac{\cos 1°+\cos 2°+\cos 3°+....\cos 44°}{\sin 1°+\sin 2°+....+\sin 44°}$$

$$= \frac{\dfrac{\sin\dfrac{44\times 1°}{2}}{\sin\left(\dfrac{1}{2}\right)^{\circ}}\times\cos\left[1+(44-1)\dfrac{1°}{2}\right]}{\dfrac{\sin 44°\times\dfrac{1}{2}}{\sin\left(\dfrac{1}{2}\right)^{\circ}}\times\sin\left[1°+(44-1)\dfrac{1°}{2}\right]}$$

$$x = \cot\frac{45°}{2}$$

Now $\cot 2\theta = \dfrac{\cot^2\theta-1}{2\cot\theta}$ $\quad \theta = \dfrac{45°}{2}$

$$1=\frac{\cot^2\frac{45^\circ}{2}-1}{2\cot\frac{45^\circ}{2}}$$

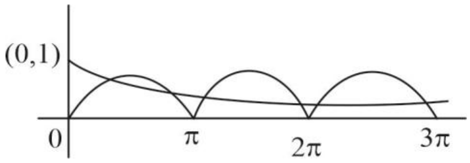
$$2x=x^2-1\Rightarrow x^2-2x=1$$

$$\Rightarrow x^2-2x+1=2$$

$$2x=x^2-1\Rightarrow x^2-2x=1$$

$$\Rightarrow x^2-2x+1=2\Rightarrow x=\sqrt{2}+1$$

5.(D)

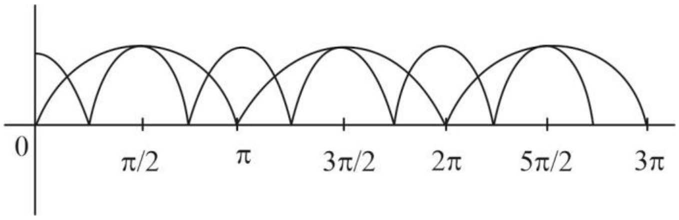


$2^{-x}=|\sin x|$

6 solutions

$m = 6$

$|\cos 2x|=|\sin x|$



$n = 9$

$n - m = 3$

6.(BC) Given equation can be expressed as

$$\pi^e(x-\pi)(x-\pi-e)+e^\pi(x-e)(x-\pi-e)+(\pi^\pi+e^e)(x-e)(x-\pi)=0$$

Let $f(x)=\pi^e(x-\pi)(x-\pi-e)+e^\pi(x-e)(x-\pi-e)+(\pi^\pi+e^e)(x-e)(x-\pi)$

$f(e)=\pi^e(e-\pi)(-\pi)>0$

and $f(\pi)=e^\pi(\pi-e)(-e)<0;$ hence given equation has a real root in (e,π)

again $f(\pi+e)=(\pi^\pi+e^e)\pi,\ e>0$

$\because\ \pi+e>\pi,$

It concludes it has a real root in $(\pi,\pi+e)$

also $\because\ \pi-e<e$

Hence $f(x)$ has two real roots in $(\pi-e,\ \pi+e)$

7.(ABD) For $x=\frac{2\pi}{7}$, then denominator in the expression of y becomes zero.

$\therefore\ y$ is not defined when $x=\frac{2\pi}{7}$

$y=\cot 4x$

for $x=\frac{\pi}{16},\frac{\pi}{32},\frac{\pi}{48}$

8.(ABCD)

$$\underbrace{\log_2 x+\log_2 x+....\log_2 x}_{n\text{ terms}}=8$$

$$\Rightarrow x^n=2^8$$

$$x^n = (256)^1 = (16)^2 = (4)^4 = (2)^8$$

9.(CD)

$$x = \left(\frac{2\cos^2 181° - 1}{\cos^2 181°}\right)\left(\frac{2\cos^2 182 - 1}{\cos^2 182°}\right) .... \left(\frac{2\cos^2 225° - 1}{\cos^2 225°}\right) .... \left(\frac{2\cos^2 269° - 1}{\cos^2 269°}\right) = 0$$

$(\because\ 2\cos^2 225° - 1 = 0)$

10.(AD) $\tan x - 3\tan 3x = \tan x - 3\frac{(3\tan x - \tan^3 x)}{1 - 3\tan^2 x}$

$$= \frac{\tan x - 3\tan^3 x - 9\tan x + 3\tan^3 x}{1 - 3\tan^2 x} = \frac{-8\tan x}{1 - 3\tan^2 x}$$

$$\therefore\ \frac{\tan x}{1 - 3\tan^2 x} = -\frac{1}{8}(\tan x - 3\tan 3x)$$

Put $x = 9°$

$$\frac{\tan 9°}{1 - 3\tan^2 \theta} = -\frac{1}{8}\left(\tan 9° - 3\tan 27°\right)$$

Similarly,

Given expression

$$= -\frac{3}{8}(\tan 9° - 3\tan 27°) + \frac{9}{8}(\tan 27° - 3\tan 81°)$$

$$+ \frac{27}{8}(\tan 81° - 3\tan 243°) + \frac{81}{8}(\tan 243° - 3\tan 729)$$

$$= -\frac{1}{8}\begin{bmatrix} 3\tan 9° - 9\tan 27° + 9\tan 27° - 27\tan 81° + 27°\tan 81° \\ -81\tan 243° + 81\tan 243° - 243\tan 729 \end{bmatrix}$$

$$= -\frac{1}{8}\left(3\tan 9° - 243\tan 729°\right)\ \left\{\tan 729° = \tan(720° + 9°) = \tan 9°\right\}$$

$$= -\frac{1}{8}(-240\tan 9° = 30\tan 9°$$

11.(B); 12.(A)

$$f_7\left(\frac{\pi}{15}\right) = \left(\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}\right)^{-1}$$

$$= \left(-\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\right)^{-1} \times \left(\cos\frac{\pi}{5}\cos\frac{2\pi}{5}\right)^{-1} \times \left(\cos\frac{\pi}{3}\right)^{-1}$$

$$= \left(\frac{-\sin\frac{16\pi}{15}}{2^4\sin\frac{\pi}{15}}\right)^{-1} \times (\cos 36° \sin 18°)^{-1} \times \left(\frac{1}{2}\right)^{-1} = 2^7$$

$$g_n(\theta) = 2\cos\left(2^n\theta\right) + 1 \Rightarrow g_3\left(\frac{\pi}{24}\right) = 2 \Rightarrow \log_{2^2} 2^8 = 4$$

SECTION 2

1.(2) $\dfrac{\sqrt{3}-1}{\sin x}+\dfrac{\sqrt{3}+1}{\cos x}=4\sqrt{2}$

$\cos x\left(\sqrt{3}-1\right)+\left(\sqrt{3}+1\right)\sin x=4\sqrt{2}\sin x\cos x=2\sqrt{2}\sin 2x$

$\Rightarrow \cos x\dfrac{\sqrt{3}-1}{2\sqrt{2}}+\dfrac{\sqrt{3}+1}{2\sqrt{2}}\sin x=\sin 2x$

$\Rightarrow \sin 15^\circ\cos x+\cos 15^\circ\sin x=\sin 2x$

$\Rightarrow \sin(x+15^\circ)=\sin 2x$

$2x=n\pi+(-1)^n(x+15^\circ)$

$(i)2x=x+15^\circ \Rightarrow x=15^\circ$

$(ii)2x=\pi-x-15^\circ$

$3x=165^\circ$

$x=55^\circ$

2.(3) Given expression is equal to $\left(\cot 70^\circ+4\cos 70^\circ\right)^2$

$\cot 70^\circ+4\cos 70^\circ$

$=\dfrac{\cos 70^\circ+4\sin 70^\circ\cos 70^\circ}{\sin 70^\circ}$

$=\dfrac{\cos 70^\circ+2\sin 140^\circ}{\sin 70^\circ}$

$=\dfrac{\cos 70^\circ+2\sin(180^\circ-40^\circ)}{\sin 70^\circ}$

$=\dfrac{\sin 20^\circ+\sin 40^\circ+\sin 40^\circ}{\sin 70^\circ}$

$=\dfrac{2\sin 30^\circ\cos 10^\circ+\sin 40^\circ}{\sin 70^\circ}$

$=\dfrac{\sin 80^\circ+\sin 40^\circ}{\sin 70^\circ}$

$=\dfrac{2\sin 60^\circ\cos 20^\circ}{\sin 70^\circ}=\sqrt{3}$

3.(4) $\sin\theta=1\Rightarrow\theta=\dfrac{\pi}{2}$

$\cos\theta=1\Rightarrow\theta=0,\ 2\pi$

$\sin\theta=0\quad \theta=0,\pi,2\pi$

$\cos\theta=0\quad \theta=\dfrac{\pi}{2},\dfrac{3\pi}{2}$

So first two factors $\sin\theta\cos\theta\left(\sin\theta-1\right)\left(\cos\theta-1\right)=0$

Will give 5 roots rest all factors of form $\left(n\sin\theta-1\right)\left(n\cos\theta-1\right)$ will give 4 each.

$5+4(n-1)=17$

$4n=16$

$n=4$

4.(4) $N = \left(\prod_{k=1}^{14} P_k\right)\left(\prod_{k=1}^{14} \log_{(k+1)}(k+2)\right) = 1.\log_2 16 = 4$

$\therefore\ (\tan x - \cot x)^2 + 4 = 4\sin^2 2x \Rightarrow \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\therefore$ Number of solution is 4.

5.(177) $\sum_{k=1}^{35} \sin 5k^\circ = \sin 5^\circ + \sin 10^\circ + ... + \sin 175^\circ$

$$= \frac{\sin 35 \times \frac{5^\circ}{2}}{\sin \frac{5^\circ}{2}} \sin\left(5 + 34 \times \frac{15^\circ}{2}\right)$$

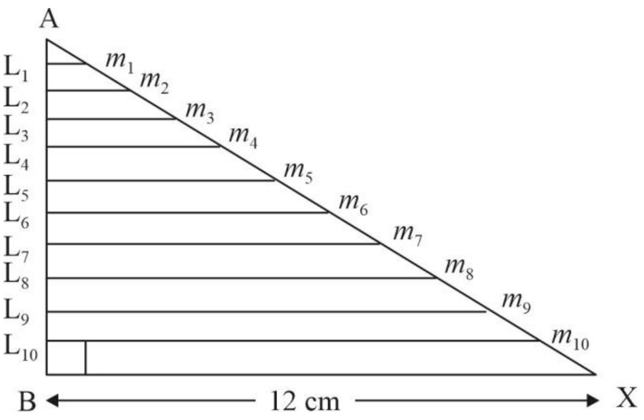
$$= \frac{\sin \frac{175^\circ}{2}}{\sin \frac{5^\circ}{2}} \sin\left(\frac{10^\circ + 170^\circ}{2}\right)$$

$$= \frac{\sin \frac{175^\circ}{2}}{\cos \frac{175^\circ}{2}} = \tan \frac{175^\circ}{2}$$

$m = 175^\circ \quad n = 2^\circ$

$m + n = 177$

6.(60)



If $AB = l$ then $AL_1 = \frac{l}{11}$

$AL_2 = \frac{2l}{11}$ and $\tan C = \frac{\ell}{12}$

$AL_{10} = \frac{10l}{11}$

$\tan C = \frac{AL_1}{L_1 m_1},\ \tan C = \frac{AL_2}{L_2 m_2}, ...\tan C = \frac{AL_{10}}{L_{10} m_{10}}$

$$L_1 m_1 + L_2 m_2 + ...L_{10} m_{10} = \frac{AL_1}{\tan C} + \frac{AL_2}{\tan C} + ... + \frac{AL_{10}}{\tan C}$$

$$= \frac{1}{\tan C}\left[AL_1 + AL_2 + ... + AL_{10}\right]$$

$$= \frac{12}{l}\left[\frac{l}{11} + \frac{2l}{11} + ... + \frac{10l}{11}\right] = 12 \times \frac{10 \times 11}{2 \times 11} = 60$$